Numeric Response Questions

Determinants

Q.1 If $D_k = \begin{vmatrix} 1 & n & n \\ 2k & n^2+n+1 & n^2+n \\ 2k-1 & n^2 & n^2+n+1 \end{vmatrix}$ and $\sum_{k=1}^{n} D_k = 56$ then find value of n.

Q.2 If A, B, C are angles of a triangle and the value of $\begin{vmatrix} e^{2i\hbar} & e^{-iC} & e^{-iB} \\ e^{-iC} & e^{2i\hbar} & e^{-i\text{Å}} \\ e^{-ik} & e^{-i\hbar} & e^{2iC} \end{vmatrix}$ is $10 - k$ then find k.

Q.3 Matrix M_r is defined as $M_r = \begin{bmatrix} r & r-1 \\ r-1 & r \end{bmatrix}$, $r \in N$. If value of $\det(M_1) + \det(M_2) + \det(M_3) + \cdots \ldots \ldots + \det\left(M_{2007)}\right)$ is k(2007) then find k.

Q.4 If $\begin{vmatrix} -a^2 & ab & ac \\ ab & -b^2 & bc \\ ac & bc & -c^2 \end{vmatrix} = \lambda a^2 bc^2$, then find the value of λ.

Q.5 Using properties of determinants, evaluate ; $\Delta = \begin{vmatrix} 265 & 240 & 219 \\ 240 & 225 & 198 \\ 219 & 198 & 181 \end{vmatrix}$

Q.6 Find the sum of infinite series $\begin{vmatrix} 1 & 2 \\ 6 & 4 \end{vmatrix} + \begin{vmatrix} 1/2 & 2 \\ 2 & 4 \end{vmatrix} + \begin{vmatrix} 1/4 & 2 \\ 2/3 & 4 \end{vmatrix} + \cdots$ .....

Q.7 If $\begin{vmatrix} x^2+x & x+1 & x-2 \\ 2x^2+3x-1 & 3x & 3x-3 \\ x^2+2x+3 & 2x-1 & 2x-1 \end{vmatrix} = Ax - 12$, then find the value of A.

Q.8 Find the number of values of k, for which the system of equation $(k+1)x + 8y = 4k, kx + (k+3)y = 3k - 1$, has no solution.

Q.9 If $\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = k(a+b+c)^3$, then find the value of k.

Q.10 If $\begin{vmatrix} x^3+4x & x+3 & x-2 \\ x-2 & 5x & x-1 \\ x-3 & x+2 & 4x \end{vmatrix} = ax^5 + bx^4 + cx^3 + dx^2 + ex + f$, be an identity in x, where a, b, c, d, e, f are independent of x, then find the value of f.

Q.11 Given $2x - y + 2x = 2, x - 2y - z = -4, x + y + 7z = 4$, then find the value of λ such that the given aystem of equation has no solution.

Q.12 If $\Delta = \begin{vmatrix} 1 & 3\cos\theta & 1 \\ \sin\theta & 1 & 3\cos\theta \\ 1 & \sin\theta & 1 \end{vmatrix}$, then find the maximum value of Δ.

Q.13 If $2ax - y + 2z = 0, x + ay + 5z = 0$ and $2x + az = 0$ have a non trivial solution then find the value of a,

Q.14 If $\begin{vmatrix} 0 & ab^2 & ac^2 \\ a^2b & 0 & bc^2 \\ a^2c & cb^2 & 0 \end{vmatrix} = 2abyc^2$, then find the value of $p + q + r$

Q. 15 If ω is a complex cube root of unity, then find the value of the determinant $\begin{vmatrix} 1 & \omega & \omega + 1 \\ \omega + 1 & 1 & \omega \\ \omega & \omega + 1 & 1 \end{vmatrix}$.

ANSWER KEY

1. 7.00	**2.** 14.00	**3.** 2007.00	**4.** 4.00	**5.** 0.00	**6.** – 10.00	**7.** 24.00
8. 1.00	**9.** 2.00	**10.** 17.00	**11.** 3.00	**12.** 10.00	**13.** 2.00	**14.** 9.00
15. 4.00						

Hints & Solutions

1. $\begin{vmatrix} n & n & n \\ n(n+1) & n^2+n+1 & n^2+n \\ n^2 & n^2 & n^2+n+1 \end{vmatrix} = 56$

$(C_1 \to C_1 - C_2,\ C_2 \to C_2 - C_3)$

$\Rightarrow \begin{vmatrix} 0 & 0 & n \\ -1 & 1 & n^2+n \\ 0 & -(n+1) & n^2+n+1 \end{vmatrix} = 56$

$n(n+1) = 56 \Rightarrow n = 7$

2. Since $A + B + C = \pi$ and

$e^{i\pi} = \cos\pi + i\sin\pi = -1$,

$e^{i(B+C)} = e^{i(\pi - A)} = -e^{iA}$ and $e^{-i(B+C)} = -e^{iA}$

By taking e^{iA}, e^{iB}, e^{iC} common from R_1, R_2 and R_3 respectively,

we have

$\Delta = -\begin{vmatrix} e^{iA} & e^{-i(A+C)} & e^{-i(A+B)} \\ e^{-i(B+C)} & e^{iB} & e^{-i(A+B)} \\ e^{-i(B+C)} & e^{-i(A+C)} & e^{iC} \end{vmatrix}$

$= -\begin{vmatrix} e^{iA} & -e^{iB} & -e^{iC} \\ -e^{iA} & e^{iB} & -e^{iC} \\ -e^{iA} & -e^{iB} & e^{iC} \end{vmatrix}$

By taking e^{iA}, e^{iB}, e^{iC} common from C_1, C_2 and C_3 respectively,

we have

$\Delta = \begin{vmatrix} 1 & -1 & -1 \\ -1 & 1 & -1 \\ -1 & -1 & 1 \end{vmatrix} = -4$

3. $|M_r| = r^2 - (r-1)^2$

$|M_1| + |M_2| + |M_3| + \ldots\ldots + |M_{2007}|$

$(1^2 - 0^2) + (2^2 - 1^2) + (3^2 - 2^2) + (4^2 - 3^2)$

$\ldots\ldots +$

$(2007^2 - 2006^2) = 2007^2$

4. L.H.S. $= a^2b^2c^2 \begin{vmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix} = 4a^2b^2c^2$

It means $\lambda = 4$

5. Use property of circular determinant

$\Rightarrow\ a + b + c = 0$

Solving $x = -9$

6. $= \begin{vmatrix} 1+\frac{1}{2}+\frac{1}{4}\ldots\ldots\infty & 2 \\ 6+2+\frac{2}{3}\ldots\ldots\infty & 4 \end{vmatrix}$

Using $a + ar + ar^2 \ldots\infty = \dfrac{a}{1-r}$

$= \begin{vmatrix} \frac{1}{1-\frac{1}{2}} & 2 \\ \frac{6}{1-\frac{1}{3}} & 4 \end{vmatrix} = \begin{vmatrix} 2 & 2 \\ 9 & 4 \end{vmatrix} = -10$

7. Put $x = 1$, we get

$\Rightarrow \begin{vmatrix} 2 & 2 & -1 \\ 4 & 3 & 0 \\ 6 & 1 & 1 \end{vmatrix} = A - 12$

$\Rightarrow 12 = A - 12 \Rightarrow A = 24$

8. $\Delta = \begin{vmatrix} k+1 & 8 \\ k & k+3 \end{vmatrix} = 0$

$(k+1)(k+3) - 8k = 0$

$k^2 + 4k + 3 - 8k = 0$

$k^2 - 4k + 3 = 0$

$(k-1)(k-3) = 0$

$k = 1, 3$

Then $\Delta_x = \begin{vmatrix} 4k & 8 \\ 3k-1 & k+3 \end{vmatrix} = 0$

$\Delta_x \Rightarrow (k^2 - 3k + 2) = 0$

$\Delta_x \Rightarrow (k - 1)(k - 2) = 0$

$k = 1, 2$

$\Delta_x \neq 0$ at $k = 3$

No. of values of k is only one.

9. put $a = 1, b = -1, c = 2$ OR

$C_1 \to C_1 + C_2 + C_3$ and taking $2(a + b + c)$ common

$$= 2(a+b+c)\begin{vmatrix} 1 & a & b \\ 1 & b+c+2a & b \\ 1 & a & c+a+2b \end{vmatrix}$$

Apply $R_2 \to R_2 - R_1, R_3 \to R_3 - R_1$

10. Put $x = 0$ both side

$$\begin{vmatrix} 0 & 3 & -2 \\ -2 & 0 & -1 \\ -3 & 2 & 0 \end{vmatrix} = f$$

$f = 17$

11. $\Delta = 0 \Rightarrow \begin{vmatrix} 2 & -1 & 2 \\ 1 & -2 & -1 \\ 1 & 1 & \lambda \end{vmatrix} = 0 \Rightarrow \lambda = 3$

12. We have,

$$\Delta = \begin{vmatrix} 1 & 3\cos\theta & 1 \\ \sin\theta & 1 & 3\cos\theta \\ 1 & \sin\theta & 1 \end{vmatrix}$$

[Applying $R_3 \to R_3 - R_1$]

$$= \begin{vmatrix} 1 & 3\cos\theta & 1 \\ \sin\theta & 1 & 3\cos\theta \\ 0 & \sin\theta - 3\cos\theta & 0 \end{vmatrix}$$

$= -(\sin\theta - 3\cos\theta)(3\cos\theta - \sin\theta)$

$= (3\cos\theta - \sin\theta)^2$

Now, $-\sqrt{9+1} \leq 3\cos\theta - \sin\theta \leq \sqrt{9+1}$

$\Rightarrow 0 \leq (3\cos\theta - \sin\theta)^2 \leq 10$

13. $\begin{vmatrix} 2a & -1 & 2 \\ 1 & a & 5 \\ 2 & 0 & a \end{vmatrix} = 0$

$\Rightarrow (a - 2)(2a^2 + 4a + 5) = 0$

14. $a^3b^3c^3 \begin{vmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{vmatrix} = 2a^3b^3c^3$

$\therefore\ p = q = r = 3$

15. $\Delta^1 = \Delta^{n-1}$

$n = 3$

$\Delta^1 = \Delta^2 = (11)^2$

$(\Delta^1)^2 = (11)^4 = 14641$